LESSONS 7-9: Coordinate Geometry

This lesson covers:

1. Cartesian system in two dimensions and point in a plane
2. Linear graphs
3. Interpretation of graphs, graphical representation of simultaneous linear equations and scatter diagrams

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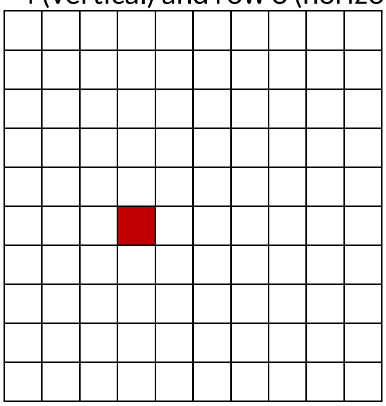
Cartesian system (Lesson 7)

1. The Cartesian system

i. Finding location of an object

The Cartesian system is used to accurately describe the position of a point, line or object. This is done using a coordinate system, which describes the vertical and horizontal position of the point, line or object.

There are many real-life examples of coordinate systems. For instance, let's say you have a ticket for a seat in a concert hall, which has 10 rows of chairs, and 10 chairs in each row. If you want to know exactly where your seat is, you could locate it by looking at which column it is in, and which row. A seat in column 4 (vertical) and row 6 (horizontal) would be labelled (4, 6) and is shown here:



A point in a plane can be found using a similar system, which we call the Cartesian system.

ii. The Cartesian plane

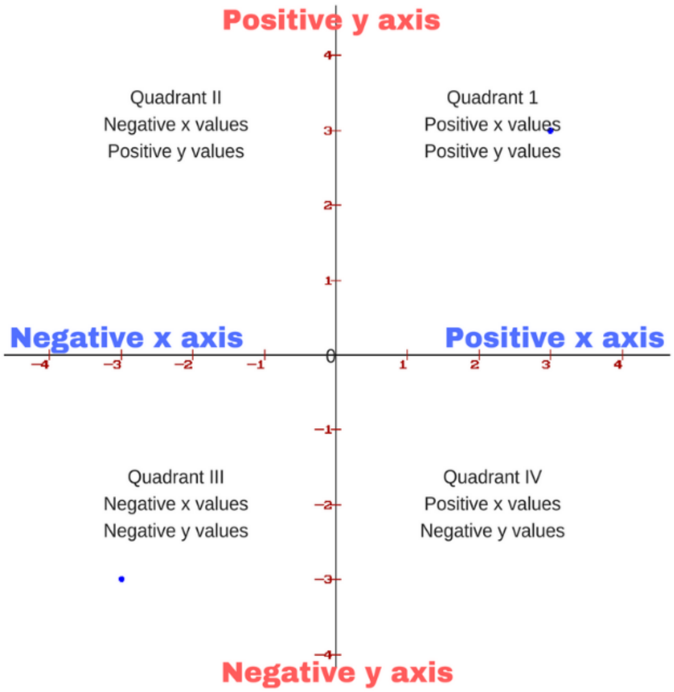
The Cartesian system divides a plane into a grid, with two perpendicular axes marked on the grid.

- The axes function like two perpendicular number lines.
- The vertical axis is called the **y-axis** and the horizontal axis is called the **x-axis**.
- The points where the axes intersect is the point where they are both equal to zero, which we call the **origin**. The origin has the coordinates (0, 0).

We can mark points on the number lines as we move away from the origin, with equal units between each point.

- On the *x*-axis (the horizontal axis), the points are positive to the right of the origin and negative to the left of the origin.
- On the *y*-axis (the vertical axis), the points are positive above the origin and negative below the origin.

The axes divide the plane into four quadrants as follows:

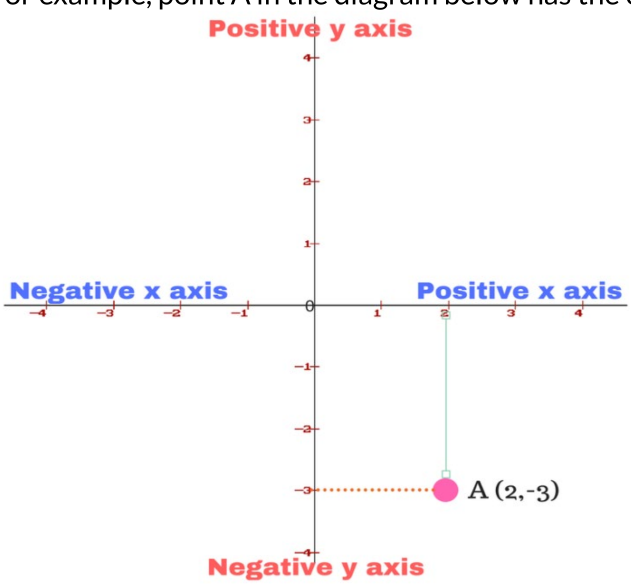


2. Point in a plane

i. Finding coordinates of a point

The location of a point in the Cartesian plane is given using an *x*-coordinate and a *y*-coordinate, which together form an **ordered pair**.

For example, point A in the diagram below has the coordinates (2, –3):

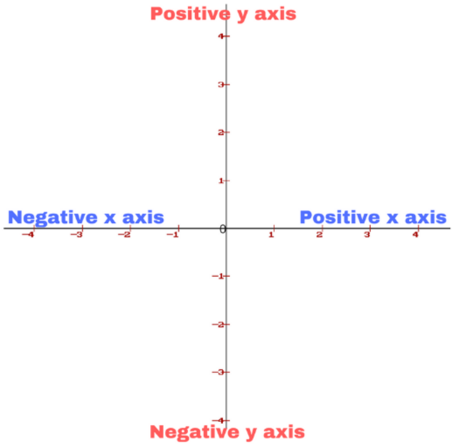


The *x*-coordinate is also called the **abscissa**. Here it is 2 and it is written first in the ordered pair. This is the distance of point A from the *y*-axis, along the positive *x*-axis.

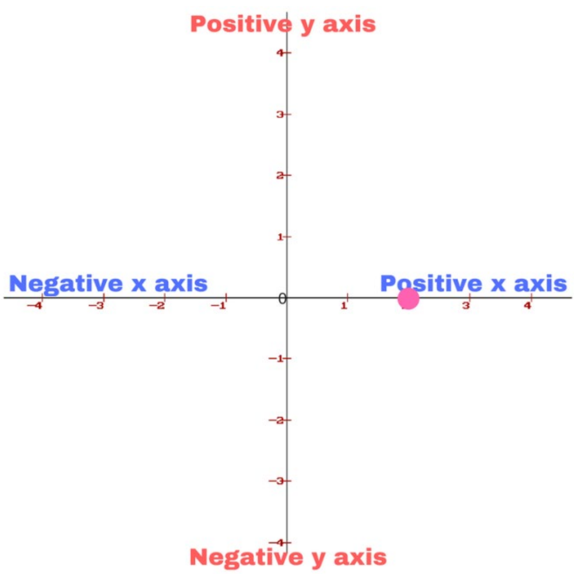
The *y*-coordinate is also called the **ordinate**. Here it is –3 and it is written second in the ordered pair. This is the distance of point A from the *x*-axis, along the negative *y*-axis.

ii. Plotting with given coordinates

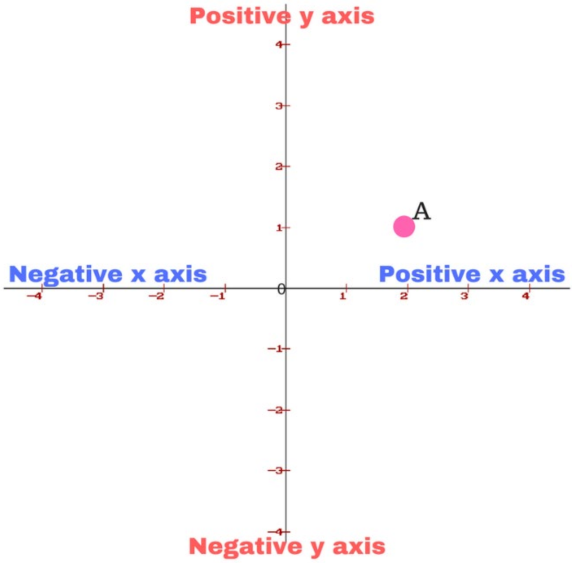
To plot a point in the coordinate plane, first start by drawing the axes.



Now you can plot the point on the plane you have drawn.
Let's plot the point A (2, 1).
2 is the *x*-coordinate and it represents the distance of A from the *y*-axis. Start at the origin (0, 0) and move 2 units along the *x*-axis, to the right.



Now look at the *y*-coordinate, which is 1. This represents the distance of point A from the *x*-axis. So start at the point you marked on the *x*-axis and move 1 unit up, along the *y*-axis. This is the point A (2, 1):



You can see that point A lies in the first quadrant, and the signs of both coordinates are positive.

Linear graphs (Lesson 8)

1. Gradient of a line

A linear graph is made up of many points in the Cartesian plane, that all lie in a straight line.
The **gradient** or slope of the line is the rate at which the *y*-value changes with respect to the *x*-value.
We can write this as:
$\text{Gradient} = m = \frac{\text{change in } y}{\text{change in } x} = \frac{y_A - y_B}{x_A - x_B}$
Note that the symbol for gradient is *m*.

Worked example
Find the gradient of the line between the points A (5, 6) and B (2, 4).

Worked solution:

$$m = \frac{y_A - y_B}{x_A - x_B}$$
$$= \frac{6-4}{5-2}$$
$$= \frac{2}{3}$$

Notes on gradient:

- If *m* is **positive**, then the gradient is positive and the straight line will slope upwards from left to right.
- If *m* is **negative**, then the gradient is negative and the straight line will slope downwards from left to right.
- A **horizontal line** is a line that is parallel to the *x*-axis. Each point on the line will have the same *y*-value, so the gradient of this line will be **0**.
- A **vertical line** is a line that is parallel to the *y*-axis. Each point on the line will have the same *x*-value, so the gradient of this line will be **undefined**, as the changes in *x*-values will be 0.
- Two **parallel lines** will have the same gradients.
- When two lines are **perpendicular** to each other, the product of their gradients will be −1.

2. The equation of a linear graph

i. General equation

The general equation of a linear graph is $y = mx + c$, where:

- *m* is the gradient,
- *c* is the *y*-value where the graph cuts the *y*-axis. This is also called the *y*-intercept.

If a line goes through the origin, then $c = 0$.

Note that this is a linear equation, which you learnt about in Unit 2.

ii. Finding the equation

When you are given the *y*-intercept and one point that lies on the graph, you can find the equation of the graph as follows:

- Start with the equation *y* = *mx* + *c*.
- The *y*-intercept is the value of *c*, so replace *c* with this value in the equation.
- Now find the gradient, *m*, using the formula $m = \frac{y_A - y_B}{x_A - x_B}$.

Worked example

Find the equation of the graph that has intercepts the *y*-axis at (0, 2) and that passes through the point (5, 7).

Worked solution

The *y*- intercept is (0, 2), so *c* = 2.

∴ *y* = *mx* + 2

Then calculate the gradient:

$$m = \frac{y_A - y_B}{x_A - x_B}$$
$$= \frac{7-2}{5-0}$$
$$= 1$$

You have found the equation for the graph: *y* = *x* + 2.

When you are given any two points that lie on the graph, you can find the equation of the graph as follows:

- First calculate the gradient, *m*, using the equation $m = \frac{y_A - y_B}{x_A - x_B}$.
- Now substitute one of the points into the equation to find the value of *c*.

Worked example

Find the equation of the line passing through points: A (2, 4) and B (5, 7).

Worked solution

First calculate the gradient:

$$m = \frac{y_A - y_B}{x_A - x_B}$$
$$= \frac{4-7}{2-5}$$
$$= 1$$

∴ *y* = (1)*x* + *c*

Now substitute one of the points into the equation to find the value of *c*:

Let's use the given point A (2, 4):

4 = 1(2) + c

4 = 2 + c

∴ *c* = 2

You found have the equation for the graph: *y* = *x* + 2.

iii. Drawing when given equation [Lesson 9]

To draw a graph when you are given the equation of the line:

- Plot the *y*-intercept from the value of *c* in the given equation.

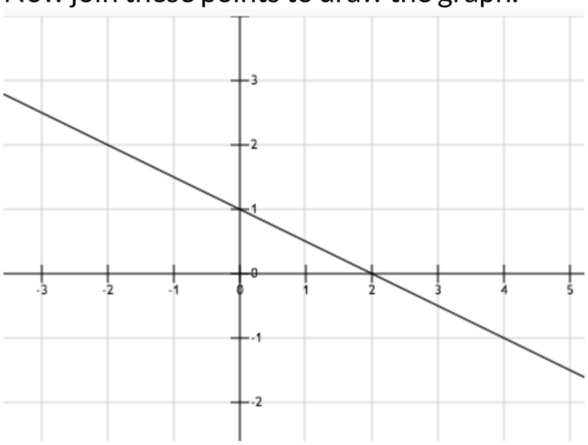
- Use the gradient to find another point on the graph.
- Join the points to draw the graph.

Worked example

Draw the line with the equation $y = -\frac{1}{2}x + 1$.

Worked solution

In the given equation, *c* = 1. Therefore the graph will cross the *y*-axis at the point (0, 1).
The gradient is $-\frac{1}{2}$. You can use this to find another point on the graph:
Start at the *y*-intercept (0, 1).
Move down one unit and 2 units to the right. You will end up at the point (2, 0).
Now join these points to draw the graph:



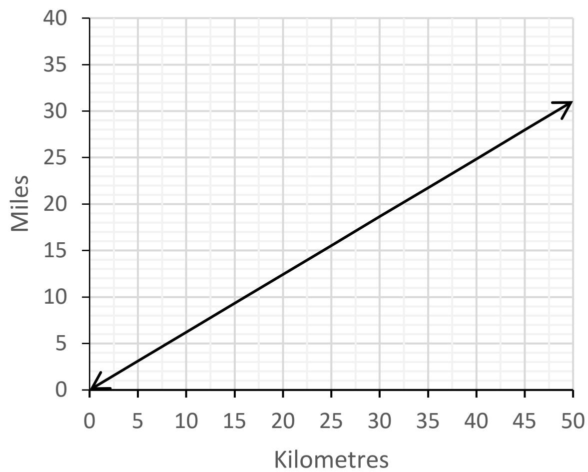
Interpretation of graphs (LM03)

1. Real-life information

Graphs tell a story, so it is important that you can interpret the information they show.

i. Conversion graphs

Conversion graphs show the rate of conversion between two quantities. For example:



You can use these graphs to do conversions between different units without having to do a calculation each time.

Worked example

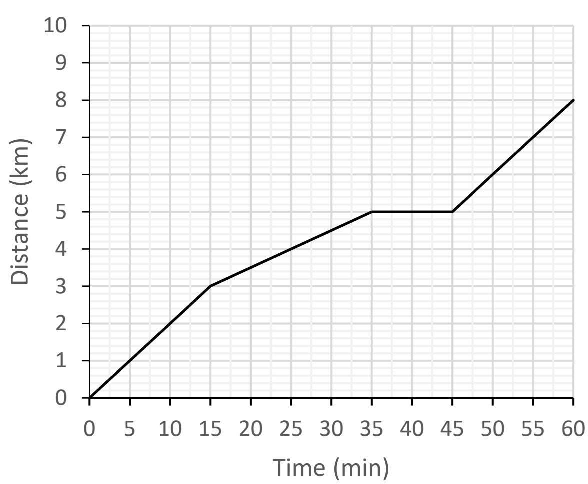
You ran 40 km. What is this distance in miles?

Worked solution

Find the distance 40 km on the *x*-axis. Now find the corresponding distance on the *y*-axis. The corresponding distance in miles is 25 miles.

ii. Travel graphs

Travel graphs show information like distance, speed or acceleration on the *y*-axis, and time on the *x*-axis. For example:



You can use these graphs to interpret the movement of a person or vehicle and to do calculations.

Worked example
Look at the graph above. It shows the distance covered by a runner over a time period of an hour.

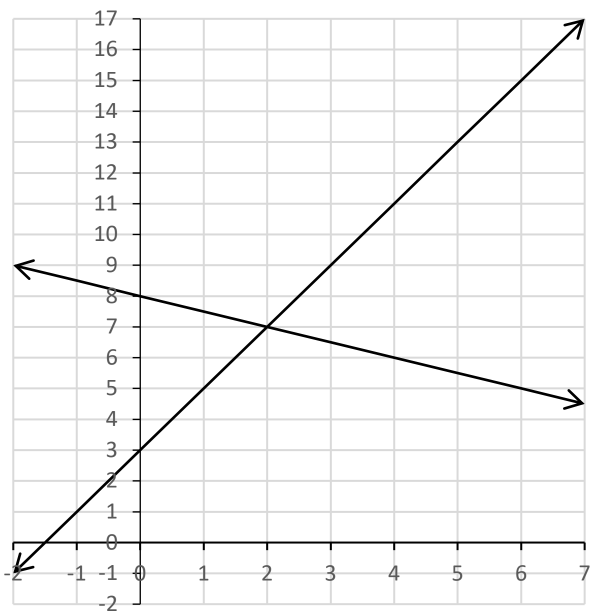
a) At what speed was the runner moving, in km/min, during the first 15 minutes?
b) Along the way, the runner stopped to have a water break. When did this break start, and how long was it?
c) What was the total distance covered in 60 minutes?

Worked solution

a) The speed was constant during this time. The runner covered 3 km in 15 minutes, so:
Speed = distance ÷ time = 3 km ÷ 15 = 0.2 km/min
b) You can see the water break on the graph as it is the section where the distance did not increase. The break started at 35 minutes and it lasted for 10 minutes.
c) The total distance covered was 8 km.

2. Graphs of simultaneous equations

When two graphs are drawn on the same set of axes, the point where they intersect represents the solution to the two equations of the graphs.
You can therefore use graphs to solve linear equations simultaneously.
For example:



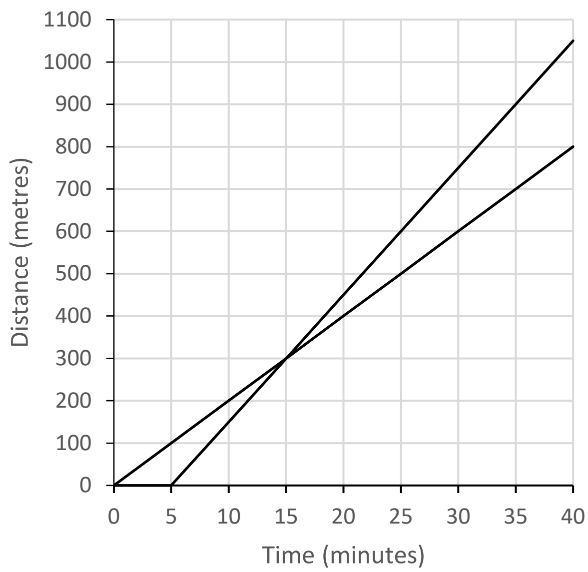
The two graphs shown here have the equations $y = 2x + 3$ and $y = -\frac{1}{2}x + 8$.
The point where they intersect is the point (2, 7). This point represents the solution to both equations.
Let's test this by substituting $x = 2$ into both equations:
Substituting $x = 2$ into $y = 2x + 3$:
$y = 2(2) + 3$
$y = 4 + 3 = 7$
Substituting $x = 2$ into $y = -\frac{1}{2}x + 8$:

$$y = -\frac{1}{2}(2) + 8$$

$y = -1 + 8 = 7$

The answer in both cases is $y = 7$. So in both equations when $x = 2$ then $y = 7$, and both lines therefore pass through the point (2, 7).

Worked example
The graph below shows the distance travelled by two friends, both starting from the same place and walking the same route. One of the friends started travelling after the other had already left.
a) At what time did the faster friend pass the slower friend on the road?
b) What distance had both friends covered when the faster friend passed the slower friend?



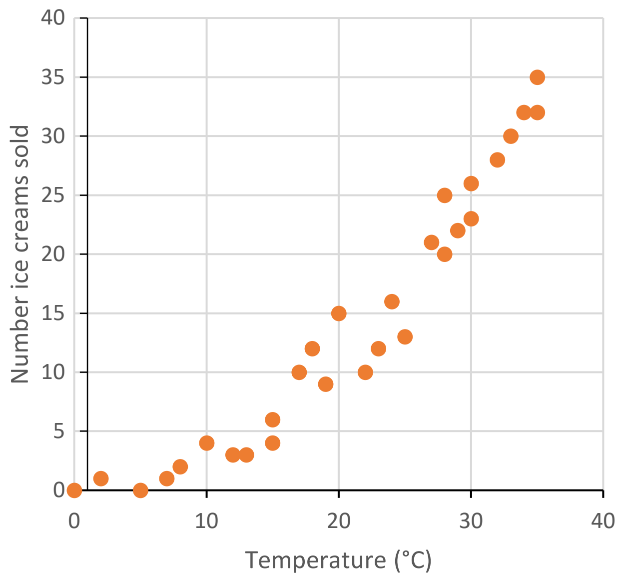
Worked solution
The point where the faster friend passed the slower friend is the point where the graphs intersect. This is the point (15, 300) on the graph.
a) The time is shown on the x-axis, so the time that had passed at this point is given by the x-coordinate of the point of intersection: 15 minutes.
b) The distance is shown on the y-axis, so the time that had passed at this point is given by the y-coordinate of the point of intersection: 300 metres.

3. Scatter diagrams

A scatter diagram is a graph that is drawn to show two independent variables. The coordinates are plotted as individual points. You can use a scatter diagram to see if there is any correlation (relationship) between the variables.

If there is a correlation you can draw a line of best fit that shows a general trend in the data:
- A positive correlation is when there is a general positive trend (sloping upwards from left to right).
- A negative correlation is when there is a general negative trend (sloping downwards from left to right).
- There is no correlation if no clear pattern can be seen in the plotted points.

Worked example
An ice cream seller wants to find out if there is a difference in the number of ice creams sold, with a change in temperature. He records the number of ice creams he sells each day and the average temperature and draws the following scatter diagram:



a) Is there a correlation between the variables? If so, describe the correlation.
b) The weather report for the following day predicts an average temperature of 5 degrees Celsius. Should the ice cream seller stay at home?

Worked solution
a) Yes, there is a positive correlation between the variables.
b) Yes, he should stay at home as it is unlikely that he will sell many ice creams.